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EFFECT OF VISCOSITY AND HEAT CONDUCTION ON THE ASCENT OF A THERMAL UNDER THE INFLUENCE OF BUOYANCY

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The ascent of a heated mass of air (thermal) in the earth's atmosphere under the influence of buoyancy is a classical problem [1]. The study [2] examined the motion of a vortex ring without consideration of its internal structure. The approximation yielded basic integral characteristics describing the evolution of a rising cloud. The motion of a heated air mass in the atmosphere was analyzed in [3] without consideration of viscosity and heat conduction. The investigations [4-7] studied convective motion of a thermal on the basis of numerical solution of two-dimensional Navier—Stokes equations with constant values of the viscosity coefficient and thermal conductivity. The present study shows how the temperature dependence of the viscosity coefficient and thermal conductivity affect the ascent of a cloud of heated gas under the influence of buoyancy.

Let a spherical region D_0 of heated gas of radius R_0 be located at the altitude H_0 in the earth's atmosphere at the initial moment of time. The gas has been heated to the temperature $T_1 > T_0$ (T_0 is the temperature of the undisturbed air). The gas pressure inside the spherical region will be assumed to be equal to the pressure in the undisturbed atmosphere at the corresponding altitude. Buoyancy will cause the heated gas, initially at rest, to rise and bring into motion new layers of the atmosphere. This is accompanied by convective and diffusive mixing of the heated and cold air, which equalizes the temperature and density of the gas inside and outside the perturbed region [8, 9].

The motion of the air during ascent of the thermal is described by the complete system of Navier—Stokes equations for a viscous compressible heat-conducting gas. This system appears as follows in a cylindrical coordinate system written in matrix form for the dimensionless variables

$$\frac{\partial f}{\partial t} + Af = F, \quad A = A_1 + A_2 + A_3 + A_4, \tag{1}$$

where $f = \begin{pmatrix} \rho \\ u \\ v \\ T \end{pmatrix}$ is the vector of the unknown variables, while the differential matrix opera-

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tors A_l, into which the operator A is broken down, are written in the form

$$A_1 = u\frac{\partial}{\partial x}E - \frac{1}{\mathrm{Re}\,\rho}\frac{\partial}{\partial x}\mu\frac{\partial}{\partial x}B_1,$$

$$A_2 = v\frac{\partial}{\partial r}E - \frac{1}{\mathrm{Re}\,\rho r}\frac{\partial}{\partial r}r\mu\frac{\partial}{\partial r}B_2,$$

$$A_3 = \begin{Vmatrix} 0 & \rho\frac{\partial}{\partial x} & 0 & 0 \\ \frac{\gamma-1}{\gamma\rho}T\frac{\partial}{\partial x} & 0 & 0 & \frac{\gamma-1}{\gamma}\frac{\partial}{\partial x} \\ 0 & 0 & 0 & 0 \\ 0 & (\gamma-1)T\frac{\partial}{\partial x} & 0 & 0 \end{Vmatrix},$$

$$A_4 = \begin{Vmatrix} 0 & 0 & \frac{\rho}{r}\frac{\partial}{\partial r}r & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\gamma-1}{\gamma}\frac{T}{\rho}\frac{\partial}{\partial r} & 0 & 0 & \frac{\gamma-1}{\gamma}\frac{\partial}{\partial r} \\ 0 & 0 & \frac{\gamma-1}{\gamma}T\frac{\partial}{\partial r} & 0 \end{Vmatrix},$$

where

$$E = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 & & & 0 \\ & 4/3 & & \\ & & 1 & \\ 0 & & & \frac{\gamma}{\mathrm{Pr}} \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & & & \\ & 1 & & 0 \\ & & 4/3 & \\ 0 & & & \gamma/\mathrm{Pr} \end{pmatrix}.$$

The components of the vector of the right sides of Eq. (1) $F = \begin{pmatrix} 0 \\ F_u \\ F_v \\ F_T \end{pmatrix}$, containing mixed derivatives and external forces in the equations of motion and the dissipative function in the energy equation, have the form

$$F_u = \frac{1}{\mathrm{Re}\,\rho r}\left[\frac{\partial}{\partial r}r\mu\frac{\partial v}{\partial x} - \frac{2}{3}\frac{\partial}{\partial x}\mu\frac{\partial rv}{\partial r}\right] - 1,$$

$$F_v = \frac{1}{\mathrm{Re}\,\rho}\left[\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial r} - \frac{2}{3}\frac{\partial}{\partial r}\mu\frac{\partial u}{\partial x} - 2v\left(\frac{\mu}{r^2} + \frac{1}{3}\frac{\partial}{\partial r}\frac{\mu}{r}\right)\right],$$

$$F_T = \frac{2\gamma\mu}{\mathrm{Re}\,\rho}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial r}\right)^2 + \left(\frac{v}{r}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial r}\right)^2 - \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial rv}{\partial r}\right)^2\right].$$

The boundary conditions for system (1) on the boundary Γ of the theoretical region $D \supset D_0$ are as follows:

$$\begin{gathered} u(x, r, t) = v(x, r, t) = 0, \ \rho(x, r, t) = \rho_a(x), \\ T(x, r, t) = T_a(x), \ (x, r) \in \Gamma, \ r \neq 0, \\ \frac{\partial u}{\partial r} = \frac{\partial \rho}{\partial r} = \frac{\partial T}{\partial r} = v = 0, \quad r = 0. \end{gathered} \tag{2}$$

At the initial moment of time (t = 0) the velocity, density, and temperature of the air in the theoretical region are given in the form

$$\begin{gathered} u(x, r, t = 0) = v(x, r, t = 0) = 0, \ (x, r) \in D, \\ \rho(x, r, t = 0) = \begin{cases} \frac{\rho_a(x)\,T_a(x)}{T_1(x, r)}, & (x, r) \in D_0, \\ \rho_a(x), & (x, r) \in D \backslash D_0, \end{cases} \\ T(x, r, t = 0) = \begin{cases} T_1(x, r), & (x, r) \in D_0, \\ T_a(x), & (x, r) \in D \backslash D_0. \end{cases} \end{gathered} \tag{3}$$

In problem (1)-(3) we introduced dimensionless variables and parameters using the following formulas (the primes were omitted in (1)-(3)):

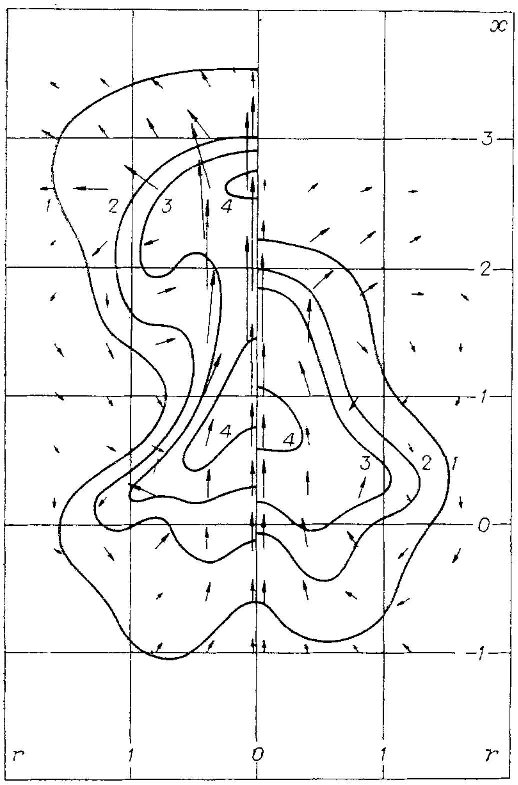

Fig. 1

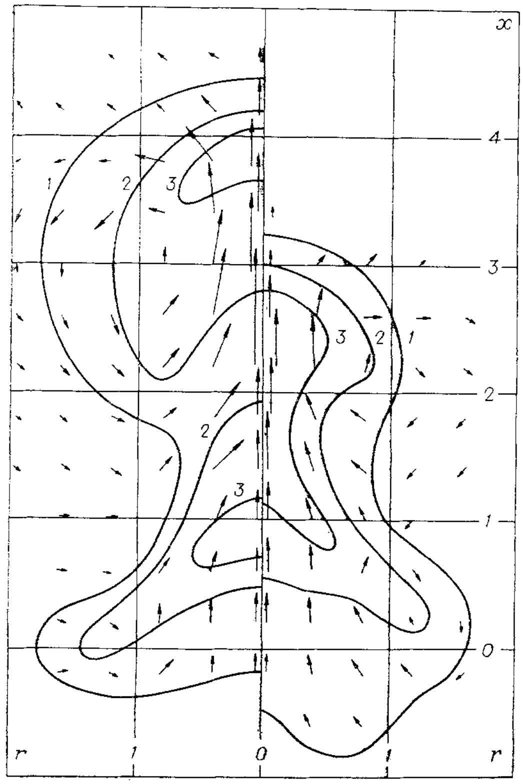

Fig. 2

$$x' = (x - H_0) R_0^{-1}, \quad r' = rR_0^{-1}, \quad t' = tt_0^{-1}, \quad u' = uu_0^{-1}, \quad v' = vu_0^{-1},$$
$$\rho' = \rho\rho_0^{-1}, \quad \mu' = \mu\mu_0^{-1}, \quad \lambda' = \lambda\lambda_0^{-1}, \quad T' = Tu_0^{-2}, \tag{4}$$
$$t_0 = \sqrt{R_0/g_a}, \quad u_0 = \sqrt{R_0 g}, \quad \mathrm{Re} = \frac{\rho_0 \sqrt{R_0 g} R_0}{\mu_0}, \quad \mathrm{Pr} = \frac{c_p \mu_0}{\lambda_0},$$

where x and r are coordinates; t is time; ρ is the density of the air; u and v are the vertical and radial components of velocity; μ and λ are the viscosity coefficient and thermal conductivity; T is temperature; $\gamma = c_p/c_V$ is the ratio of the specific heats of the air; the index a pertains to quantities in the undisturbed atmosphere, while the index 0 pertains to quantities at the height H_0.

The temperature dependence of absolute viscosity was taken in the form [10] $\mu = \mu_0(T/T_0)^{\omega}$.

Problem (1)-(3) was solved by the numerical method of breaking a problem down according to physical processes and spatial directions [11]. The difference scheme was constructed relative to changes in the increments of the grid functions on adjacent time layers in accordance with the recommendations made in [12].

Numerical solution of problem (1)-(3) yielded space–time dependences of the fields of density, velocity components, and temperature for different ω.

As was shown in [8], the formation of a vortex ring from a spherical region of light gas is due mainly not to the motion of the gas comprising the thermal but to flows of denser surrounding layers of air. Here, however, the flow of initially undisturbed atmospheric air into the region of reduced pressure on the wake of the rising thermal depends considerably on the mean rate of ascent of the thermal. The mean velocity is determined by the difference in the densities of the gas inside and outside the heated region and, as the completed calculations showed, also depends on the values of the viscosity coefficient and thermal conductivity of the gas. In the case of high values of viscosity, friction between the more heated central layers of the thermal and its less heated peripheral layers impedes an increase in the mean rate of ascent of the cloud. This is illustrated by Figs. 1 and 2, which show isotherms and the velocity field for times t = 0.5 and 0.75, respectively. The right sides of the figures show isotherms and the velocity field with $\omega = 0.7$, while $\omega = 0$ for the left sides. The isotherms correspond to the following temperatures: curve 1) $1.05T_0$; 2) $2.0T_0$;

3) $3.0T_0$; 4) $5.0T_0$. The directions and lengths of the arrows indicate the directions and magnitudes of the local velocity of the gas. It is apparent from Figs. 1 and 2 that despite the qualitative agreement between the patterns of air motion, there are important quantitative differences which can be attributed to allowing for the temperature dependence of the viscosity coefficient and thermal conductivity. Allowing for these dependences enhances the effect of viscosity on air motion in the thermal and leads to a decrease in its rate of ascent. Here, there is a corresponding slowing of the transformation of the spherical volume filled with heated gas into a vortex ring.

The completed calculations show that viscosity has a significant effect on the convective rise of a heated air mass and on the rate of formation of the vortex ring. In numerically modeling the convective motion of a heated air mass, it is necessary to make specific allowance for the temperature dependence of the viscosity coefficient and thermal conductivity of the gas.

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